CHAPTER I

FUNDAMENTALS

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General description

The current method of position determination is piloting. This is estimating the position by means of our known course, speed and sailing time. For long journeys offshore we will need to "refresh" our ***estimated*** *position* or ***Pe***. This updating is the purpose of celestial navigation and is as follows.

Before making any measurements we should:

1. Calculate what the height above the horizon of a given celestial body at a given time would be if observed from our estimated position at a given time, this is the calculated height ***Hc*** above the horizon.
2. Calculate what the ***azimuth Zn***, of the given celestial body at the given time would be if observed from our estimated position at the given time.

after that we will measure:

The height above the horizon of the given celestial body at the given time. This is measured from our real position.This is the ***measured height Hm*** above the horizon. With the difference between Hc and ***Hv*** (1) we shall be able to perform the necessary correction on our estimated position, the difference is called the ***intercept*** Δh.

At daytime the celestial body will be the sun, the described method will have to be repeated several times to obtain different intercepts.

At night, we have numerous visible celestial bodies to choose from, we shall repeat the method for each selected star at approximately the same time. With the different intercepts obtained, we will be able to construct the ***position triangle*** on a ***chart***.

(1) Hv is equal to Hm including a correction for refraction and other errors

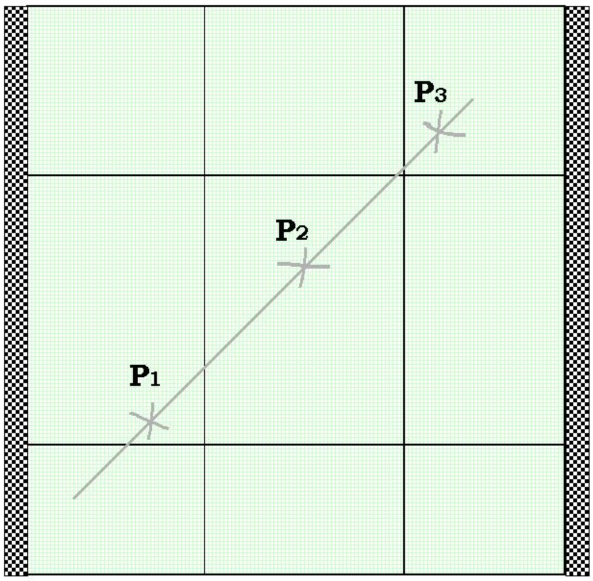
Estimation of the Position

For readers who are unfamiliar with piloting a brief recapitulation is given. Good seamanship always requires the knowledge of the following elements:

- The position from which we leave, i.e. the harbour.

- Our average speed and direction, in which currents and tides are taken in account.

As we plot those elements on a chart (1) we are able to estimate our position at a given moment of our journey.



On the figure P_0 is our initial position, P_1 our estimated position after for example one hour sailing, P_2 our estimated position after, for example, 3 hours sailing. That is because the distance covered is proportional to our speed. Evidently the precision of our estimation decreases with the elapsed time. That is why we need to complete this plot by transforming our estimated position to real positions.

(1) Chart is a nautical term for map

Geometrical approach of Hc and Zn

The calculation of Hc and Zn is a problem of solving a ***spherical triangle*** of which two sides and the enclosed angle are known. This is very similar to solving a plane triangle with the classical formulas of trigonometry.

The spherical triangle we have to solve is called the ***navigation triangle***. This navigation triangle is fully defined by three points, which are our estimated position Pe, ***the earth projection*** , or ***EAP*** of the celestial body or ***CB*** and the north pole ***N***.

We will demonstrate that the dimensions of the navigation triangle are proportional or equal to Hc and Zn.

Next paragraph begins with general definitions and properties and is meant as a base on which we will build.

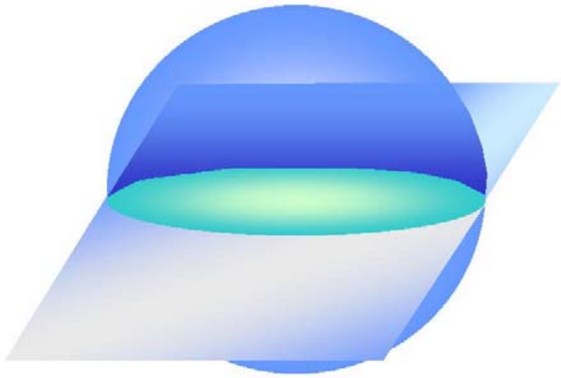
Applying these definitions to the earth sphere and navigation triangle will make definitions and demonstrations more concise.

These paragraphs are meant to explain which elements are calculated and measured and on which geometrical properties they are based.

The calculations will be performed after we have introduced the miscellaneous co-ordinates systems.

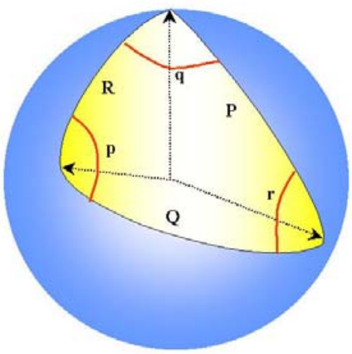
General Geometrical definitions and properties

Definition of a great circle



A great circle of a sphere is the cross section of a plane, which goes through the centre of the sphere with the sphere surface. This cross section is a flat circle in space, which has the same centre and radius as the sphere. A great circle is fully defined once two of its surface points are known. The shortest way between two points on a sphere surface is a segment of the unique great circle going through these two points of the surface. On earth all the meridians and the equator are great circles.

Definition of a spherical triangle

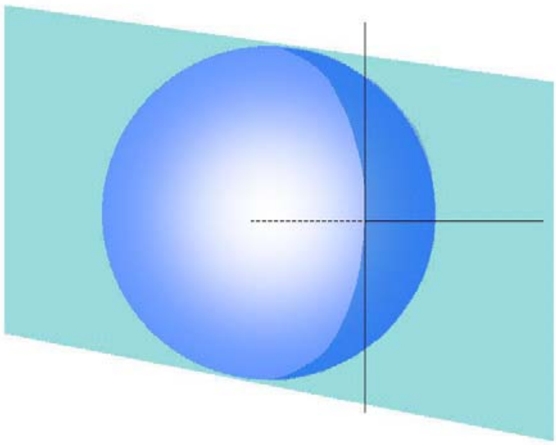


Suppose three arbitrary point's P, Q, R on the surface of a sphere, when we connect these points with the segments of great circle on which they respectively lay we obtain a spherical triangle. The lengths of the spherical triangles sides are equal to the respective lengths of the great circle segments. The angles of a spherical triangle are the angles that the tangent lines of the great circle segments make with each other in the points P,Q,R.

Definition of the horizontal and vertical planes as seen from the earth's surface

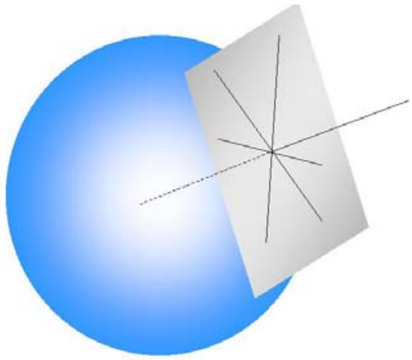
A human observer is infinitely small compared to the earth's sphere so we are allowed to consider him as a point on the earth's surface. The only portion of the earth's sphere that our "point observer" is able to see confounds itself with the tangent plane through his feet on the earth surface. This gives him the impression that the earth is flat. We consider the tangent plane as the horizontal plane of the observer. The connection line between him and the centre of the earth is his vertical axis. Any plane that contains this axis is a vertical plane through the observer.

Definition of a vertical plane as seen from space



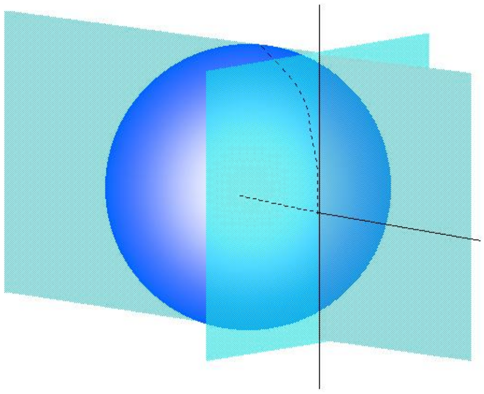
Any plane that contains the connection line between an arbitrary point of the earth's surface and the centre of the earth is vertical on the earth's surface in that arbitrary point. As each one of these planes contain the centre of the earth they each define a great circle (see def. of a great circle) Each plane which contains the North Pole and which is vertical in an arbitrary point of the earth surface is called the ***Meridian plane*** trough that arbitrary point. The great circle defined by such a plane is called a ***meridian***.

Definition of a horizontal plane as seen from space



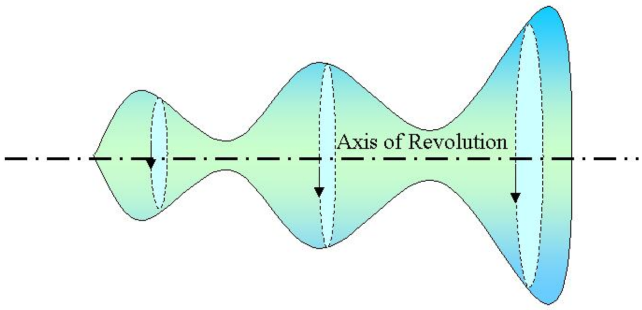
A plane is horizontal in an arbitrary point of the earth's surface when it is perpendicular on the connection line between that point and the centre of the earth. So it is the tangent plane to the earth in that point. All lines of these planes in that point are tangent to the earth's sphere. So each line is tangent to a particular great circle in that arbitrary point.

Intersection between vertical and horizontal plane



Consider an horizontal and a vertical plane in an arbitrary point of a sphere. The intersection line of both planes is the tangent line to the great circle defined by the vertical plane in that arbitrary point. As the line is part of the horizontal plane it is tangent to a particular great circle through that point. As it is also part of the vertical plane it is tangent to the particular great circle defined by that vertical plane

Definition of a Surface of revolution

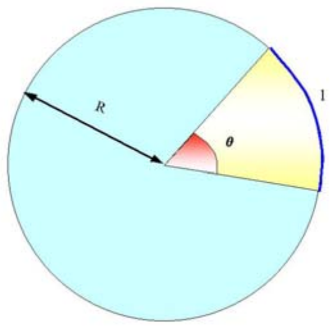


A surface of revolution is generated by revolving a curve about a line called the axis of revolution.

Properties

Each cross section of the surface with a plane perpendicular to the axis of revolution is a flat circle, called parallel circle (1). The centre of a parallel circle, **c,** is the point of intersection between the axis and the perpendicular plane to this axis. The axis of revolution is an axis of symmetry in each plane which contains the axis of revolution. Examples of surfaces of revolution are spheres, cones, cylinders etc.

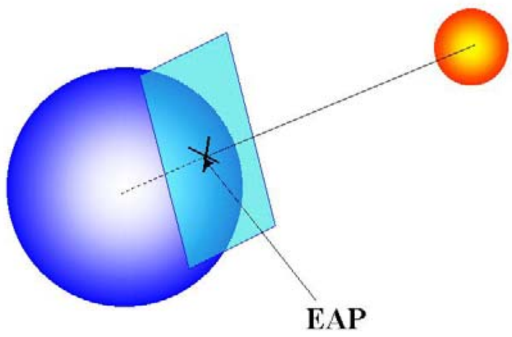
Relation between the arc length of a circle and the described angle



The arc length l of an arbitrary circle segment is directly proportional to the angle θ on which the segment lays (see figure). The proportionality factor only depends of the chosen angle units. Nautical units are defined so that the arc length and the described angle have the same value when the angle is expressed in minutes of a degree.

(1) On earth the meaning of parallel circle is restricted to the parallel circles which are generated around the North south axis.

The Earth projection of a celestial body

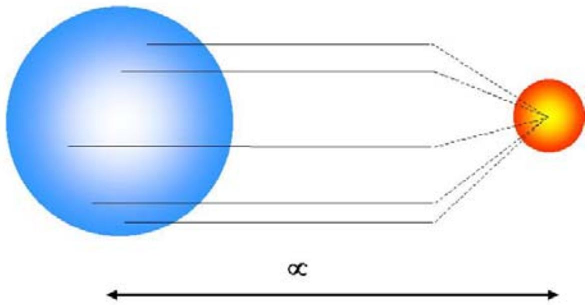


When we connect the centre of the earth and the centre of a celestial body (CB)with a straight line, the intersection of this line with the earth's surface is a unique point, this point is the earth projection or ***EAP*** of the celestial body.

The EAP of the sun is always situated in the inter-tropical zone, i.e. between tropic of Cancer and tropic of Capricorn. That is why there is no shadow on these places at certain times of the year.

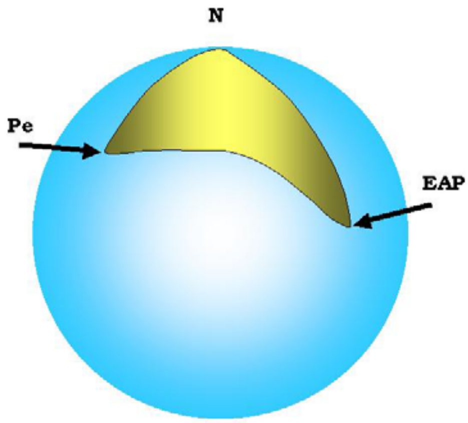
The EAP of the polar star nearly coincides with the North Pole. The location of the EAP is reported in the Nautical Almanac and is a reference point for calculations as a function of date & time.

Property of connection lines



Every line connecting an arbitrary point of the earth's surface with the centre of the CB is parallel to the line connecting EAP with the centre of the CB. Inversely any line through the earth's surface which parallel to a connection line is the unique connection line through that arbitrary point. This is due to the fact that all these lines intersect in the centre of the CB which is situated at an infinite distance from earth. This distance is infinite in comparison to the earth's radius. Lines that intersect at infinite are considered in geometry as being parallel.

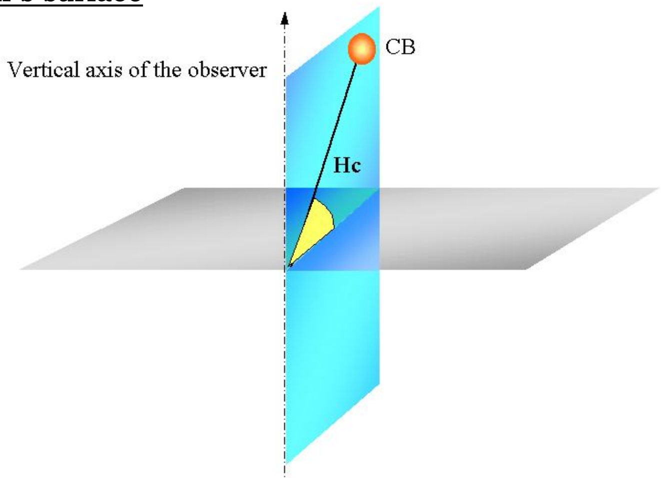
The Navigation triangle



The navigation triangle is a spherical triangle on earth formed by the North Pole N, Pe and EAP. The lengths of the arcs N EAP and N Pe and the angle between these arcs in N are known values, as the used co-ordinates' system is referenced relatively to the meridian planes through these positions. With these parameters we are able to compute the arc length between Pe and EAP and the angle between the arc N Pe and the arc Pe EAP. This arc length is proportional to the height above the horizon while the angle in Pe defines the azimuth. The formulas used for these computations are the haversine and sine formulae for spherical angles.

The calculated height above the horizon.

Hc as seen from the earth's surface



Suppose a human observer situated at Pe measures the height above the horizon of the CB. The measured angle is situated in the vertical plane containing both the observer and the CB. The angle Hc is the angle between the connection line Pe CB and the intersection line between this unique vertical plane and the horizontal plane.

Space and plane view of the vertical plane

The cross-section of the unique vertical plane with the earth's surface is the great circle through Pe and EAP, which is a side of the navigation triangle.

Demonstration

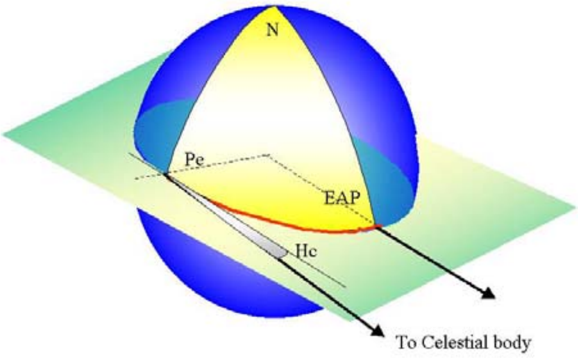
The cross-section is a great circle, because the plane is vertical in Pe (1)

Pe lays on the cross-section due to the proposition

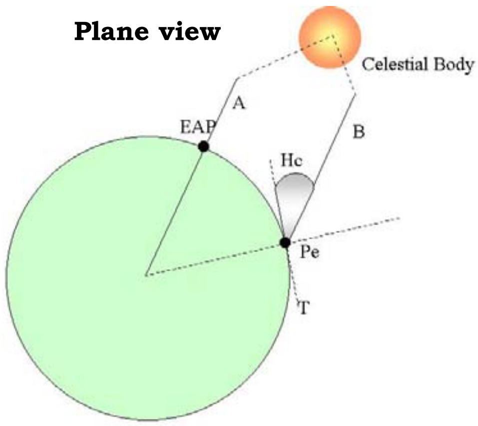
As CB and the centre of the earth lay (1) in the vertical plane their connection line lies in the plane too. The common point of this line with the earth's surface is EAP(2), so EAP is a point of the cross-section too.

This makes that the great circle is fully defined because two of its earth surface points are known, i.e. Pe and EAP. (3)

Space view



Plane view



As Hc is measured in the unique vertical plane, we can find Hc in the plane view

Demonstration

The first side of Hc is the intersection between the horizontal and vertical planes in Pe, this side corresponds to the tangent T in Pe (4) to the circle in fig 1.z

The second side is the connection line Pe CB. This line corresponds to line B in Pe, parallel to the line A. Line A is the CB EAP connection line.

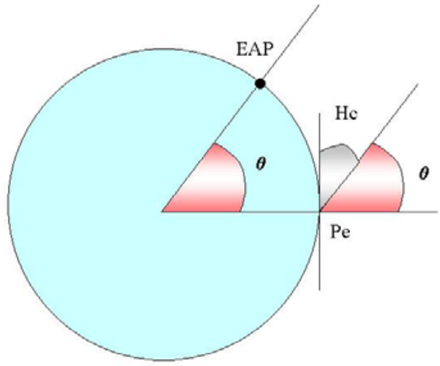
As all these connection lines between points of the earth's surface with the CB are parallel to each other (5) so line B is the connection line Pe CB.

The angle Hc was defined in space view as being as the angle between these lines.

(1) See definition of vertical plane
(2) See definition of EAP
(3) See definition of great circle
(4) See intersection of horizontal and vertical planes
(5) See definition of connection line

Properties of the plane view of Hc

Consider the angle θ defined by the great circle segment EAP Pe in the plane view of Hc.



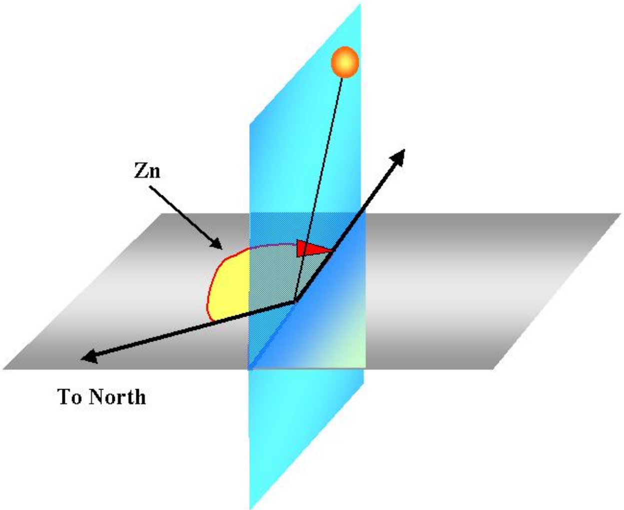
The plane view establishes that Hc and θ are complementary (1) angles, as Hc has respectively one side parallel and one side perpendicular to a side of θ.

This means if we compute the distance between Pe EAP in nautical units (2) we find the angle θ. Subtracting θ from a right angle gives Hc as result.

This establishes that solving the navigation triangle for the side Pe EAP in function of the two other sides and their enclosed angle has as final result Hc. The formula used is the haversine formula. We will be able to use it once we know the co-ordinates' system.

(1) Two angle are complementary when their sum is a right angle
(2) See relation between arc length en described angle

The Azimuth of a Celestial body



Consider two vertical planes trough the observer in Pe, the first is the meridian plane in Pe, the second plane contains Pe and CB. The angle that the intersection lines of these vertical planes make with the horizontal plane through the observer is the azimuth of the celestial body in Pe. (1)

The azimuth is the angle in Pe of the navigation triangle

Demonstration

As the intersecting lines are the tangent lines to the great circles N Pe and Pe EAP cut respectively by the vertical planes though N Pe and Pe CB (2)

As we defined the angle of a spherical triangle as the angle between the tangents this proves the proposition.

Computation of Zn

This establishes that solving the navigation triangle for the angle in Pe in function of the arcs Pe EAP and Pe N and the angle in N gives us Zn.

The formula for the computation of Zn is the sinus formula for sperical triangles

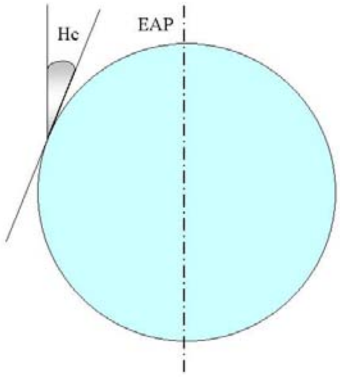
For its parametrisation, we need also to introduce a co-ordinate system.

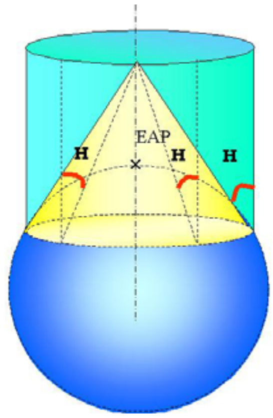
(1) This definition is truncated, as we did not take the orientation of the angle in account.
(2) We proved in plane view of Hc that the Pe CB and the Pe EAP vertical planes are identical.

The Position circle

All points of the earth's surface that are located on equal distance from EAP have the same Hc.

Visual demonstration





Consider the surface of revolution generated by revolving the plane view of Hc around the connection line EAP CB.

The result is the earth sphere for the circle, a cylinder for the connection line Pe CB and a cone for the tangent in Pe.

We can easily verify that the generating lines of the cylinder and cone intersect two by two at the same angle Hc, this is due to the symmetry of a surface of revolution.

All these points form a circle around EAP which is called the ***position circle***. Inversely for each Hc there is a corresponding position around EAP.

Very important remark

We can also verify that when Hc increases Pe is nearer to EAP which means that the radius of the position circle decreases.

Inversely if Hc decreases Pe is farther from EAP which means that the radius of the position circle increases. The use of the intercept is based on this property.

The measured and the real height above the horizon

The measured height Hm above the horizon of a celestial body is the height above the horizon of the CB we measure effectively from our real position.

This height Hm however contains errors which are:

-An instrumental error due to the inexactitude of our instrument

-A diffraction error due to the diffraction of the atmosphere

-An error because we do not measure from sea level

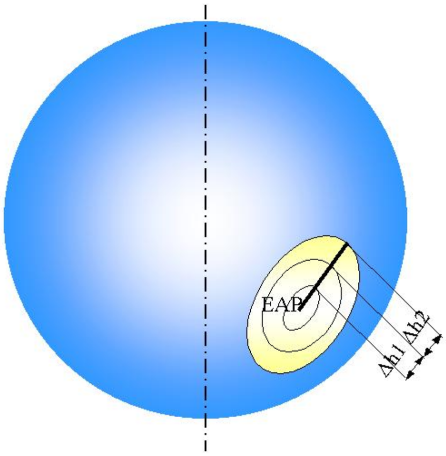
These errors are tabulated as correction factors on Hm in the nautical almanac.

Applying these correction factors on Hm will give us the ***real height Hv*** above the horizon. This real height is a height we would have measured exactly from the water surface on a planet earth without atmosphere with a perfect instrument.

The intercept Δh

The algebraic difference between the real height and the calculated height is the intercept Δh. The intercept is at the same time a difference in height and a difference in distance. As we subtract heights the terms containing the right angles are dropped. To compute the intercept, we take the real height as the calculated height this value is free of diffraction and the other errors.

Visual illustration of the calculations

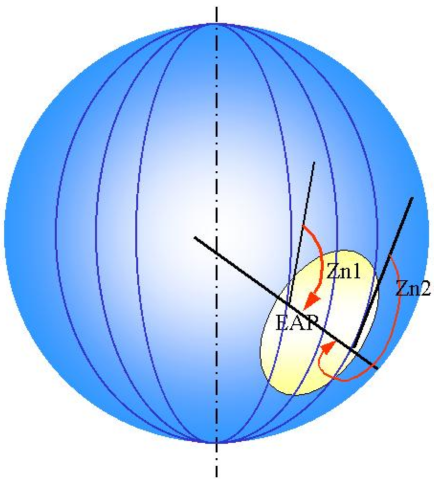


We have seen that points with the same Hc form a circle around EAP. For each Hc there is a corresponding position circle. So each Hc is represented by a concentric position circle around EAP.

By calculating Hc we determine on which of the concentric circles Pe is situated. By subtracting Hc from a right angle we obtain the radius of the position circle, when the radius is expressed in nautical units.

With the intercept Δh, which is the difference in distance between two of these position circles, we find the position circle on which we are really situated.

The calculation of the azimuth determinates which angle Zn that radius makes locally with the meridian in Pe.



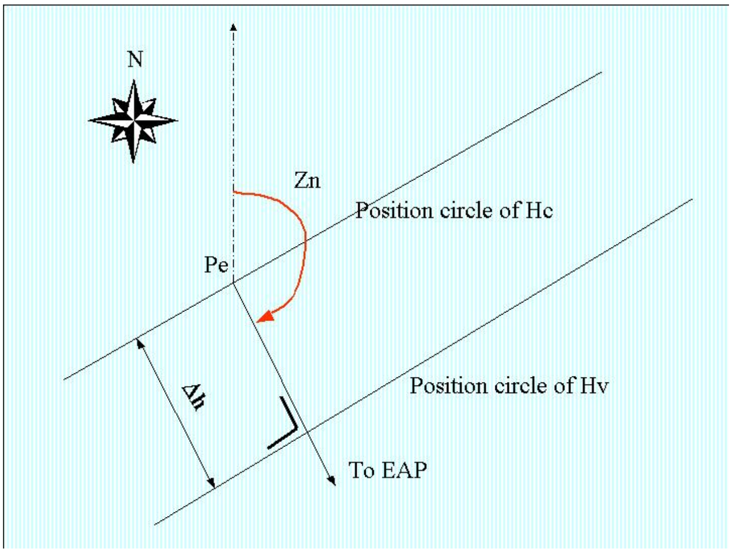
We note that Zn is different on the same radius for each meridian. That is the reason why we calculate Hc and Zn for a given Pe and that we don't calculate Pe for a given Zn and Hc.

Plotting Hc , Δh and Zn on a chart

As charts represents only a small area of the earth's surface only a part of the radius and only a segment of the position circle can be represented.

So the position circle segment becomes the ***position line***, this coincides with the tangent line to the position circle in Pe.

This is a good approximation as this circle has a huge radius compared to the distances represented on the chart.



To plot the position we draw a line in Pe that makes an angle Zn with the north. This is a segment of the position circle's radius, which is directed towards the EAP. Perpendicular to this line we draw a straight line in Pe, which is a segment of the position circle corresponding to Hc.

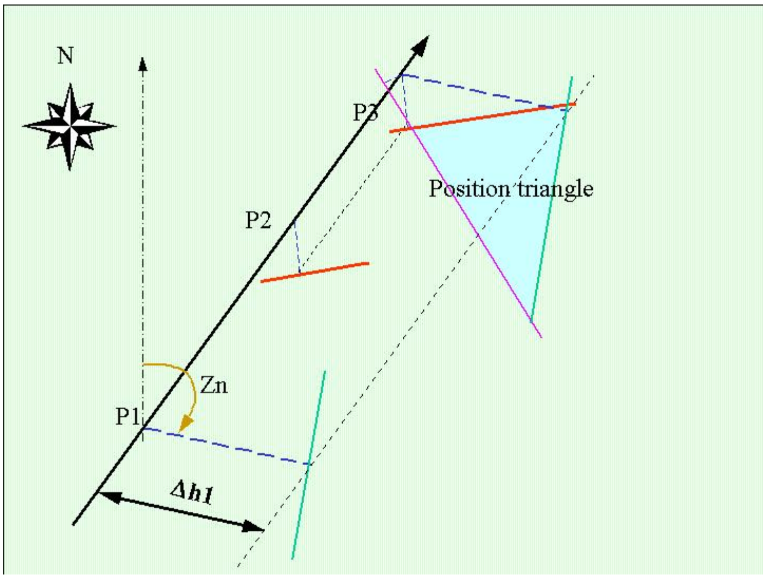
On a distance Δh from Pe we draw a parallel line to the position line in Pe. This is the position line corresponding to our real position.

In this example the real height is greater than the calculated height so the real position is nearer to the EAP. Thus we have drawn the distance Δh towards EAP, Inversely we would have drawn this line away from EAP if Hv was smaller than Hc.

This information is contained in the algebraic sign of Δh. In practice we draw immediately the position line Hv without drawing the position line belonging to Hc.

The position triangle

Let us go back to our chart on which we estimated our position at tree different moments of the day. For each estimated position we computed the intercept Δh and the azimuth Zn.



We plot these as described in the previous paragraph on the chart. Each of the tree position lines in P1, P2 and P3 represents the collection of points on which we are possibly located at respectively hour1, hour2 and hour3.

By transferring the position line belonging to P1 parallel to our course over a distance covered between P1 and P3, the transferred line is one side of the position triangle. We do the same for position P2 but for a distance covered between P2 and P3. The position line belonging to P3 has not to be moved.

These two transferred position lines and the position line P3 form a triangle, which is the ***position triangle***.

Our real position at hour3 is situated in the area of the position-triangle. The size of this area will decrease when the precision of our estimations increases. In the most ideal case the position-triangle becomes one point as the position lines intersect in that point.