CHAPTER II
THE COORDINATES SYSTEM
AND FORMULARIES

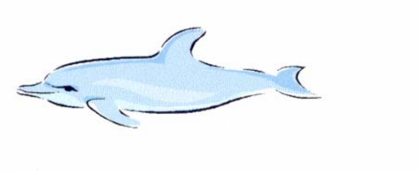
THE CO-ORDINATES SYSTEMS AND FORMULARIES

In previous section we had a purely geometrical approach in order to describe the properties of the navigation triangle so that we know what we compute.

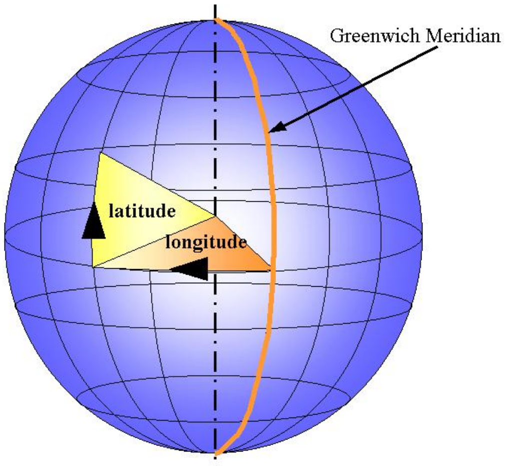
In this chapter we are only interested in how we perform the computations.

This will take the following steps:

- Establish the ***terrestrial*** co-ordinates system and describe the position of Pe,
- Establish the ***equatorial*** co-ordinates system and determine the position of EAP,
- Give the general formulas for spherical triangles,
- Convert the positions of N, Pe and EAP to point of the navigation triangle,
- Agree on a sign convention in order to make the correct arithmetical operations,
- Apply the general formulas for spherical triangles to the navigation triangle,
- Examine special configurations of the navigation triangle,
- Enumerate the different type of times.



The terrestrial co-ordinates system



The terrestrial co-ordinates system is referenced relatively to the equator and the Meridian of Greenwich.

The location of a point on the earth's surface is fully described by two values which are it's ***latitude*** and ***longitude***. This co-ordinate system is used for describing Pe.

Construction:

The North-South axis is the axis about which the earth rotates about itself in 24 hours. Consider the planes perpendicular on the North-South axis.

Each cross-section with the earth's surface defines a ***parallel circle***. The only parallel circle that is also a great circle is the ***equator***. The equator cuts the earth in two parts, the Northern Hemisphere and the Southern Hemisphere.

Each plane that contains the North-South axis is a meridian plane; each of these planes defines a great circle. Each of these great circles is cut in two half circle by the North-South axis. These half circles are called the ***meridians***. The Meridian trough Greenwich divides the earth in two equal parts which are the Western and Eastern Hemisphere.

Definition of the latitude

The latitude is the oriented angle measured along the local meridian from the equator towards the local parallel circle.

The latitude takes always the name of the hemisphere on which the position we want to refer to is situated. Its absolute value is always within the range of 0° to 90°.

So the Latitude of the equator is 0° and these of the North and South Pole are respectively at 90° N and 90° S

The standard abbreviation for latitude is ***l*** For some calculations we will use the complement of the latitude, this is the co-latitude

Definition of the longitude

The longitude is the oriented angle measured along the equator from the meridian of Greenwich towards the local meridian circle we want to refer to.

The longitude is named West or East from Greenwich and its absolute value is always within the range of 0° to 180°.

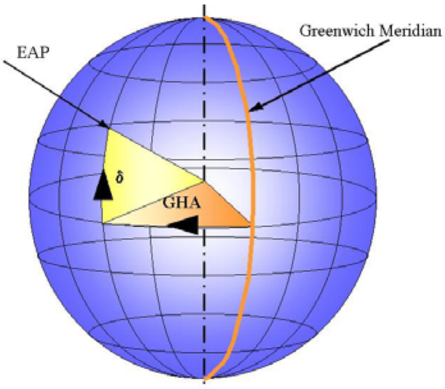
So the longitude of Greenwich is 0° and that of Guam in the Pacific Ocean is 180°, that of Washington is 30°W and that of Peking is 73°E of Greenwich. The standard abbreviation for longitude is ***g.***

Remark

We remark that the definition of latitude is based on purely physical references of the earth such as the equator and the axis of daily rotation.

For the longitude however as all the meridians are geometrically identical a particular meridian is needed as reference. The choice of the Greenwich meridian is based on historical grounds. This is totally arbitrary from a navigational point of view.

The equatorial co-ordinates system

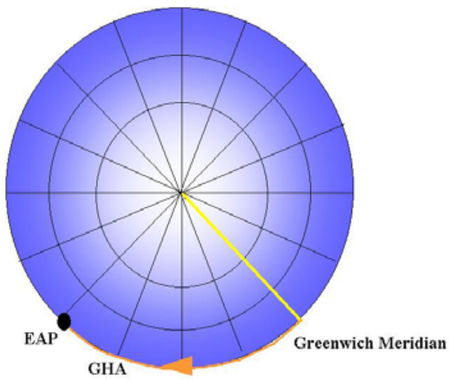


The equatorial co-ordinates system is referenced relatively to the equator and the Meridian of Greenwich. This co-ordinates system is used for describing the EAP of a celestial body. The location of the EAP on the earth's surface is fully described by two values, which are the ***declination*** and the ***Greenwich hour angle***. Both are tabulated in the Nautical Almanac.

Definition of the declination

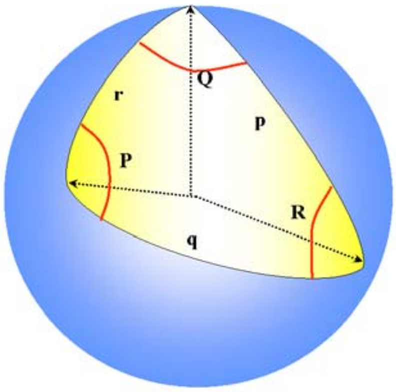
The declination of the EAP is equal to the latitude on which EAP is situated. The declination takes the name of the hemisphere on which EAP is situated and its absolute value is always within the range of 0° to 90°.The standard abbreviation for declination is δ.

Definition of the Greenwich hour angle



The Greenwich hour angle is the oriented angle measured along the equator from the meridian of Greenwich in western direction (clockwise when seen from above the North Pole) towards the local meridian on which the EAP is situated. Its value varies from 0° to 360° in one day. The standard abbreviation for the Greenwich hour angle is ***GHA***.

General formulas for spherical triangles



Nomenclature

We refer to a side opposite to a given angle with the same letter, the side takes a small letter while the angle takes a capital, hence in the spherical triangle side Q is opposite to angle q.

Formulas

For every spherical triangle the following basic formulas are valid

The cosine formula (1):

$$\cos p = \cos r \cos q + \sin r \sin p \cos P$$

The sine formula (2):

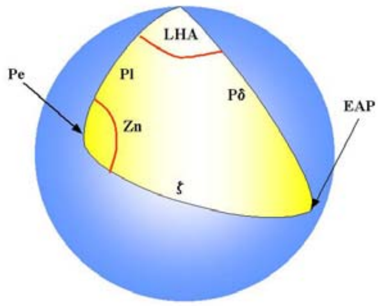
$$\frac{\sin R}{\sin r} = \frac{\sin P}{\sin p} = \frac{\sin Q}{\sin q}$$

In order to make manual computations easier the ***haversine*** function is defined:

$$\text{Hav}\ \alpha = \frac{1 - \cos\alpha}{2}$$

(1) The complete proof of these formulas is given in the appendix, knowledge of it has minor or no importance

Formulas for the navigation triangle



In order to apply the formulas (1) & (2) we need to convert the positions Pe and EAP in sides and angles of the navigation triangle. We will find the formulas for the navigation triangle by substitution of the converted values in the formulas (1) & (2)

The data to convert

From the position Pe : the latitude **l** and the longitude **g**

From the position EAP: the declination δ and the Greenwich hour angle **GHA**

The converted data

To the side **P1=90°-l**	the distance to the North Pole along the meridian of **Pe**
To the side **Pδ=90°-δ**	the distance to the North Pole along the meridian of **EAP.**
To the angle **LHA=GHA-g**	the local hour angle **LHA**

The data to compute

The side **ζ=90°-H**	the complement of the height above the horizon

Substitution

In formula (1)

cos ζ	= cos Pl	x cos Pδ	+ sin P1	x	sin Pδ	x cos LHA
cos (90°-H)	= cos (90°-l)	x cos (90°-δ)	+ sin(90°-l)	x	sin(90°-δ)	x cos LHA
sin H	= sin l	x sin δ	+ cos l	x	cos δ	x cos LHA

The formula for computations with a pocket calculator:

sin H = sin l x sin δ + cos l x cos δ x cos LHA

The same formula in haversine shape for computations with Nories tables:

Hav (90°-H)=Hav (l-δ) + cos l x cos δ x Hav LHA

The sign convention

In order to make correct algebraic differences we need a sign convention.

For the difference 1-δ:

If latitude and declination have the same name we subtract both values

If latitude and declination have opposite names we add both values

The sign of the result has no importance as Hav (**l**-δ) = Hav (δ-**l**), by convention, however we will use the positive difference as Hav is tabulated for positive angles only.

Examples

l = 30° 00' 00" N and δ = 16° 00' 00" S, hence **l**-δ=30°-(-16°)=30°+16°=46°

l = 15° 00' 00" N and δ = 20° 00' 00" N, hence we take δ-**l** = 20°-15° = 5°

For longitudes

A longitude which is named WEST takes a POSITIVE sign

A longitude which is named EAST takes a NEGATIVE sign

This is due to the fact that GHA is oriented the same as western longitudes

Examples:

g = 5° 15' 01" W hence g= 5° 15' 01"

g = 16° 19' 15" E hence g = -16° 19' 15"

For GHA

GHA is not a named value and has always a positive sign

For LHA

First apply formula : **LHA=GHA-g**

If the result is negative start again with : **LHA=GHA-g+360°**

The result must remain positive, as we will also need LHA for looking up the azimuth in the tables.

The Azimuth

By substitution in formula (2) we obtain

cotg Zn cosec (90°-l)= cotg (90°-δ) cosec LHA – cotg LHA cotg (90°-l)

or

cotg Zn sec l= tg δ cosec LHA – cotg LHA tg l (3)

Sign rules according ABC tables

|cotg LHA tg l| = |A|

The sign of A can only be N or S, opposite to l if 90°<LHA<270°, equal to l if not

|tg δ cosec LHA| = |B|

The sign of B can only be N or S and always the same as δ.

C=A+B

This is an algebraic sum and so takes the name of the greatest part and can only be N or S

cotg Zn = C cos l

The sign of Az is a combined sign which can only be SW, SE, NW, NE. The first part N or S is the sign of C, the second is E if 180°<LHA<360° and W if not.

The Azimuth is then:

Zn= Az	if NE
Zn=180°-Az	if SE
Zn=180°+Az	if SW
Zn=360°-Az	if NW

The values of ABC and Az are tabulated in "Nories Nautical Tables" in the "ABC tables".

Pocket calculator method

We bypass the ABC step, by giving an opposite algebraic sign to respectively N and S, apply the formula (3) and reconvert the sign to N or S, and add E or W and convert Az to Zn according to previous.

The Sidereal Hour Angle SHA

The **GHA** is only tabulated for a restricted number of celestial bodies as **GHA** is a value which varies from second to second. Tabulating **GHA** from minute to minute for each known star would give voluminous tables. The SHA on the contrary is nearly invariable for each star. From the SHA we can easily calculate the GHA for each star. We just need to add SHA to the GHA of point Aries. SHA is always a positive value between 0° and 360°. The symbol of Point Aries is γ

We compute the GHA of a star with the formula :

$$GHA = GHA\gamma + SHA$$

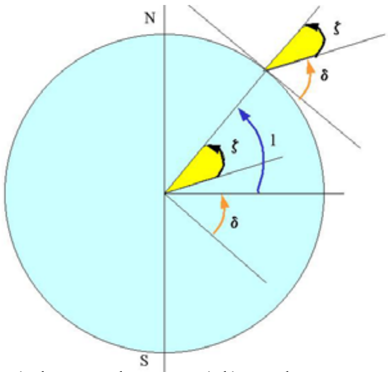
Remark on GHA

Here above GHA is always expressed in degrees but it can also be expressed in hours and minutes as it varies in one day from 0° to 360° due to the Earth's rotation. For this we use the table "convert arc to time" of the Nautical Almanac.

Special configurations of the navigation triangle

The height at meridional passage

At noon the LHA is equal to zero and the navigation triangle is thus reduced to a line. The great circle in which we measure the height above the horizon coincides with the local meridian of the observer.



On the picture we construct the height at the meridional passage which is:

$$H = 90° - (l - \delta)$$

We obtain the same result by setting LHA to zero in formula (1).

sin H = sin l x sin δ + cos l x cos δ x cos 0

sin H = cos (l-δ) or H= 90° - (l-δ)

This observation has to be made when the sun or another celestial body culminates physically in the sky. For the sun this occurs at the local noon.

Amplitudo calculation (1)

Is nothing else than an Azimuth calculation for a height equal to zero. This occurs at dawn and twilight. For this we need to solve the navigation triangle for its side Pδ and set H to 0. Substitution in the general formula (1) for spherical triangles gives:

sin δ = sin l x sin 0 + cos 0 x cos l x cos Az

sin δ = cos l x cos Az

Which gives as final result:

$$\cos Az = \frac{\cos l}{\sin \delta}$$

(1)This calculation is used for compass correction.