CHAPTER IV

CHARTS AND TRACKS

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A chart is a plane representation of the Earth's surface on which we are able to plot our sailed course and read eventually our covered distance. As a sphere is not a developable surface as for example a cone several projections can be used to represent its surface.

In this chapter we will discuss the ***Mercator projection*** and the ***Gnomonic proje***ction and their respective ***distortions***.

The Mercator projection is used for the construction of the ***Mercator chart*** and the ***middle latitude*** chart. The Mercator chart is used for plotting ***loxodromic courses*** and the middle latitude chart for plotting our estimated position. The ***Gnomonic chart*** is used for plotting ***orthodromic courses***.

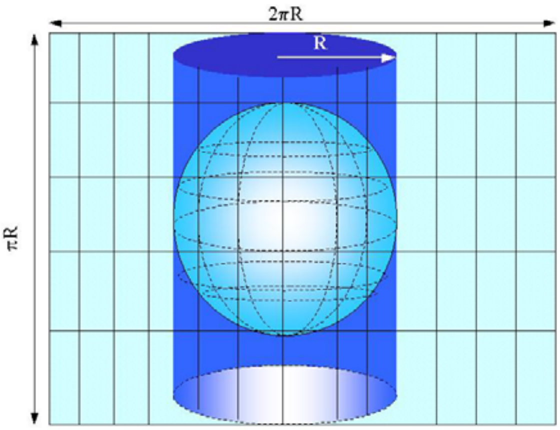
In addition we will see how to calculate a loxodromic distance, a middle latitude distance and an orthodromic distance and when we use one of the three methods.

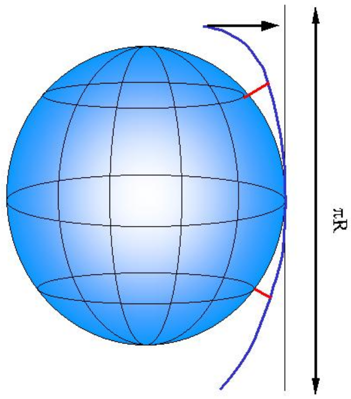
The Mercator chart

We sail a ***loxodrome*** when we keep a constant compass course i.e. when we cross each meridian with a constant angle. On the Mercator chart each straight line represents a loxodrome. The angle we have to sail for going from one point to another can be read directly on the chart. The distance between the points is determinated by comparing of the line segment and the latitude scale, as we know that one minute of a degree in latitude is equal to one nautical mile.

When the distance becomes too important we will calculate the loxodromic distance and course, as measuring directly on a chart becomes too inaccurate. The calculation method for a loxo-dromic distance and course however is based on the construction of the Mercator Chart.

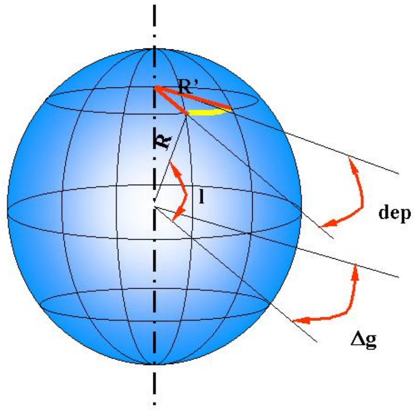
Construction





The earth's surface is projected on a cylinder which is tangent to earth at the equator. The cylinder is developed to a rectangle of $2\pi R$ x πR, respectively the length of the equator and the meridians. Hence each parallel is stretched to a straight line of $2\pi R$ and all meridians to a straight line of $2\pi R$ long. The projection is so that the parallels and meridians intersect at 90°. The chart is not yet ready as it still has an unacceptable ***distortion*** due to the variable ***departure***.

The departure



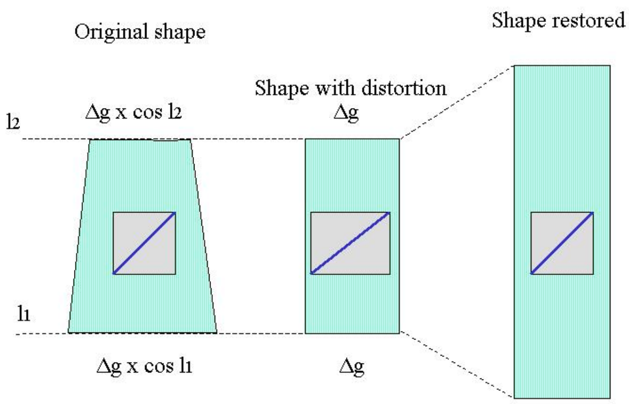
The departure is the distance between two meridians measured along a parallel. As the meridians converge at the Poles the departure decreases at higher latitudes. The figure above shows the relation between latitude and departure:

Dep= R' x Δg= R cos l x Δg

If Δg is expressed in minutes of a degree the departure in nautical miles is:

$$\boxed{\textbf{Dep} = \textbf{cos l x } \Delta\textbf{g}}$$

The distortion

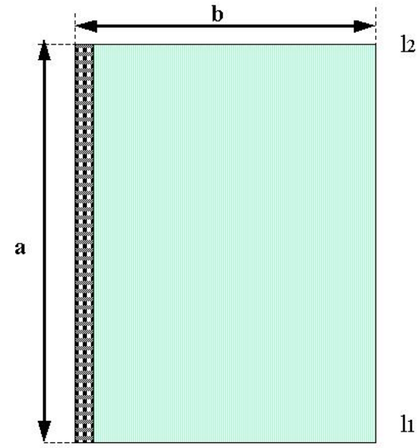


As we stretched each parallel circle to the length of the equator the meridians on the chart are all at a distance Δg from each other at any latitude. With this projection a square should become a rectangle. We see that the angle of the diagonal changed so would a course. Giving a supplementary stretch to each meridian dependent on the latitude reduces the distortion. The amount to stretch called ***Meridional Parts***, increases at higher latitudes.

Reduction of the distortion

The nautical mile is by definition equal to the arc-length of 1 minute of a degree of latitude. On a distortion free chart the distances in any direction are measured with the minute of the latitude scale.

Suppose a rectangle with dimensions a x b representing an area of the earth surface. The boundaries of the rectangle are parallel circles at the latitude l_1 and l_2 and meridians at longitude g_1 and g_2.



In order to reduce distortion we need to determine the correct ratio **a/b**.

As the length **a** has to be proportional to : $(l_2-l_1) \times 60'$

and the with **b** has to be proportional to : **Dep**$=(g_2-g_1) \times \cos l_1 \times 60'$

Hence the ratio is : $\frac{a}{b} = \frac{(l_2-l_1) \times 60'}{(g_2-g_1) \times \cos l_1 \times 60'}$

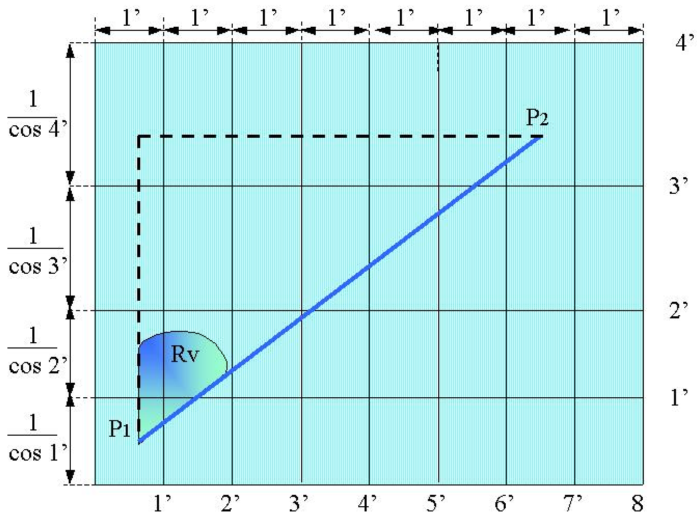
And if we choose $(l_2-l_1) = (g_2-g_1)$ the ratio becomes : $\frac{a}{b} = \frac{1}{\cos l_1}$

On the represented area the distortion is zero at l_1 and increases with the latitude as we have been using cos l_1 in the ratio. The ratio shows we have to stretch the meridians dependently of the latitude and that the stretch increases with the latitude.

The precision of the obtained chart decreases with the size of the represented area. Therefore greater areas will be subdivided in smaller rectangles.

[Meridional Parts]

When we construct a world chart we first project the earth surface with a cylindrical projection, we divide the surface in rectangles of 1’ latitude by 1’ longitude. On each rectangle we reduce the distortion by stretching the part of the meridian with the ratio discussed in the previous paragraph. The Meridional parts are the sum of these ratios by increments of latitude minute, hence they express at which distance of the equator each parallel has to be drawn. The Meridional parts are tabulated in Nories tables per minute from 0°0’ to 89°59’.



They are obtained using the following formula: $\sum_{n=1}^{5400} \frac{1}{\cos(n \times 0°01')}$

The symbol for a meridional part : ***lc***

Distance and course of a loxodromic track

Solving the plane right-angled triangle with hypotenuse P_1P_2 on the figure above gives exactly for the course over the ground (route vraie):

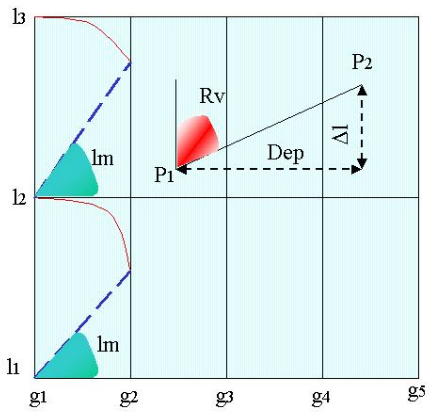
$$\textbf{tan Rv} = \frac{(g_1-g_2)}{(lc1-lc2)}$$

And approximately, as the meridional parts are discontinuous at each latitude, for the length of the track:

$$\textbf{M} = \frac{(l_1-l_2)}{\cos Rv}$$

The Middle latitude Chart

The Middle latitude Charts are based on the same principle as the Mercator chart. They only differ in the used latitude increments. On the middle latitude chart the navigator knows which increments to use. He draws them daily by plotting the estimated position. As only small areas of the globe are represented much greater increments can be chosen.



Construction

- Draw the meridians equidistantly

- Draw an oblique line at an angle equal to the mean latitude of the area at:

$$lm = \frac{(l_1 + l_2)}{2} \text{ and } lm = \frac{(l_2 + l_3)}{2}$$

- Report off the length of the oblique line on the vertical axis
- Be sure that for each obtained rectangle $\Delta l = \Delta g$, for example $l_3 - l_2 = g_4 - g_3$

Distance and Course

On the figure above we solve the right-angled triangle and we obtain for the course over the ground, the departure is taken at the mean latitude of P_1 and P_2:

$$\tan Rv = \frac{Dep}{\Delta l}$$

And for the length of the track:

$$M = \frac{(l_1 - l_2)}{\cos Rv}$$

In Nories tables all the solutions for rectangular triangles are tabulated, the possible entries are Δl, Δg, Rv, lm and Dep.

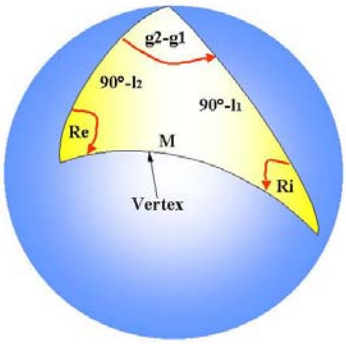
Orthodromy

We are sailing an orthodromic track between two points when our track coincides with the segment of the great circle connecting these two points. In the chapter "Fundamentals" we saw that this track is also the shortest distance between two points on earth. On the contrary to a loxodromic course, the course of an ortodromic track varies continuously. We will sail according to an orthodrome when the distances become too important. The loxodromic course however easier to compute becomes too inaccurate.

Remark

GPS devices will give you an orthodromic track between two way-points and the momentary course to steer hence it is more convenient to compute the loxodromic track in order to sail a constant course.

Calculation of the distance and Course



In order to find the formulas for the initial course R1 and M we need to solve the spherical triangle for which:

- The sides are the co-latitudes l_1 and l_2 of both positions and the distance M
- The angles are the initial course R1 and the ending course R2.
- The top angle is the difference g_2-g_1 in longitude of both points.

In order to find the formulas for intermediate courses Ri we need to calculate the ***vertex*** of the great circle we sail. The vertex is the point of the great circle where latitude is maximum.

Formulas for Orthodromy

The distance:

In order to compute the orthodromic distance between two points we use the cosine formula for spherical triangles given in the appendix "Formularies".

$\cos p = \cos q \cos r + \sin q \sin r \cos P$

After substitution of $r=90°-l_2$, $q=90°-l_1$, $p=M$, $P=\Delta g$ we find the formula for the distance M:

$\cos M = \cos(90°-l_1)\cos(90°-l_2) + \sin(90°-l_1)\cos(90°-l_2)\cos(g_2-g_1)$

Or

$$\boldsymbol{\cos M = \sin l_1 \sin l_2 + \cos l_1 \cos l_2 \cos \Delta g}$$

The initial course

We find the initial course by substitution in the sine formula for spherical triangles by substitution in sine formula $\sin R1 = \sin \Delta g \sin(90°-l_2)/\sin M$

Or

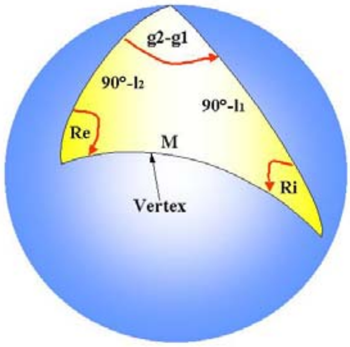
$$\boldsymbol{\sin R_1 = \frac{\sin \Delta g \cos l_2}{\sin M}}$$

The end course

The end course is obtained by the same substitution but we solve with respect to R_2

$$\boldsymbol{\sin R2 = \frac{\sin \Delta g \cos l_1}{\sin M}}$$

Computation of the vertex



ln order to compute intermediate courses of an orthodromic track we must compute the position of the vertex of the track. The course at the vertex is 90° or 270° by definition. The vertex, the pole and the initial position form a right-angled spherical triangle enclosed in our original spherical triangle formed by RI, R2 and the pole. Of this triangle we know the initial course RI the initial latitude Il and the course Rv=90° at the vertex.

The latitude of the vertex

We find the latitude Iv of the vertex by substitution in the cosine formula for spherical rectangular triangles. **ln** sin q =sin Q sin r we substitute q =90° -Iv, Q=R1 sint =90°-11 or sin (900-1v)= sin R1 sin (90-11) which gives

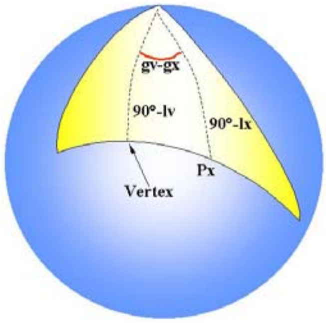
$$\boxed{\cos lv = \sin R1 \cos l1}$$

The longitude of the vertex

ln the formula cos P = cotg r tg q we substitute P = gv-gl and r=90o-11 and q = 90-lv or cos (gv -gl)= cotg (900 -Il) tg (900- Iv) which gives:

$$\boxed{\cos(gv-g1) = tg\ l1\ cotg\ lv}$$

Intermediate positions and courses



Once we know the vertex we can compute relatively to the vertex the intermediate positions and courses of the point Px of our track. We substitute gl and Il by the intermediate longitude and latitude gx and lx in the vertex formula for longitude. Hence we find the intermediate latitude lx. Which gives

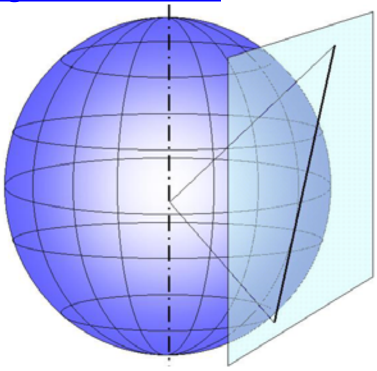
tg lx= cos (gv-gx) tg Iv

tg lx= cos (gv-gx) tg Iv

ln the vertex formula for latitude we substitute 11 by lx and RI by Rx in order to find the intermediate course Rx, which gives:

$$\sin Rx = \frac{\cos lv}{\cos lx}$$

The gnomonic chart



ln order to visualise orthodromes the gnomonic chart is used. This kind of chart is also a flat representation of the earth's surface. When we connect two points on the chart by a straight line this straight line represents the orthodrome between these two points. So we can read directly aIl the intermediate positions on the chart without calculation.

Method

Depending on which si de of earth we want to represent we choose a tangent plane to earth. We connect by a straight line each point of the earth's surface with the earth's centre. The intersections of these lines with the tangent plane is the gnomonic projection of each point of the earth's surface.

How orthodromes become straight lines

As orthodromes are great circles they all are intersections of planes going through the earth 's centre. Hence all the connection lines from the surfaœ points to the centre lay in the same plane. So their gnomonic projection is the line of intersection of the plane through the earth's centre with the tangent plane. Inversely we can find such a plane for each straight line on the chart as a plane is determined solely by a straight line and a point.