CHAPTER V

NUMERICAL EXAMPLES

NUMERICAL EXAMPLES

In this chapter we will give numerical examples of each type of computation i.e. loxodromic and orthodromic route computations, traverse tables, plotting and computing of the geographical position. An example of position computation with the ***HO249 tables*** is included.

The instrumental errors, which are the chronometer error ***c***, the sextant error ***i***, the height of the boat in order to find the "***dip***" correction are given, because they are specific to each instrument and ship

The Nories tables

The computations are performed by the means of "***Nories tables***". The ability to use the tables is imperative in order remain independent from electronic devices. Pocket calculators give the same result, possible differences are due to rounding. The Nories tables remain forever valid and are available at any ship-chandler.

Logarithms in Nories tables

In Nories tables logarithms are given the following:

log 0,5 = ~~1~~,69897 instead of log 0,5 =-0,30103, where ~~1~~,69897=-1+1-0,30103

this kind of shape is meant to eliminate subtractions on decimals.

Example:

log 316,23	2,50000
log 0,5	-0,30103
	2,19897

becomes

log 316,23	2,50000
log 0,5	~~1~~,69897
	2,19897

The Nautical Almanac

In the nautical almanac the coordinates δ, GHA or SHA of sun, moon, planets and stars are tabulated on a daily base. This book is only valid for one year and is sold by any ship-chandler.

The HO249 tables

This almanac gives the coordinates of the stars including their hc and Zn in function of the latitude. These tables remain forever valid at the exception of yearly correction page. The tables are freely available and downloadable on the site of the National Imagery Agency (NIMA).

PROBLEM 1

On the first of October 2001 a boat is steering a course of 212° at a speed of 5 knots. Her estimated position at 9 am deck time is 35° 19'N; 15° 17,2 W.
For measurements the sextant was hold by a man of 1,7 m tall and 32 years old on a deck at 1,3 m above the waterline. The chronometer is running 5s too fast. The deck time is directly derived from the chronometer time. Regrettably the sextant has an index error of +2 minutes.
The following altitudes of the sun were noted at the respective deck times :

09h02m06s	*34° 35,0'*
10h03m15s	*43° 42,3'*
11h00m00s	*49° 27,6'*
Meridional passage	*51° 20,8'*

Determine the position of the boat at meridional passage using the Nautical Almanac, the Nories tables. Use your pencil, parallel rulers and a pair of compasses. Don't use your pocket calculator please, except for verification.

Solution :

1° summarise the data

2° compute the track with the traverse tables from Nories.

3° determine the basic parameters LHA and δ for each position

4° compute heights, azimuths and intercepts of each position

5° Plot each position line from the final position.

6° Measure Δl and Dep on the plot and convert Dep to Δg.

7° Apply Δl and Δg on the final position in order to obtain the real position.

Summarised data

DATE	Time Zone	Dip	Rv	speed	c	i	l_1	g_1
01/10/2001	GMT-1	3m	180°+32°=212°	5kn	-5s	+2'	35° 19,0'N	15° 17,2' W

Observation at Local Time	Measured height
09h02m06s	34° 35,0'
10h03m15s	43° 42,3'
11h00m00s	49° 27,6'
Meridional pass. = +/-12h00	51° 20,8'

Local Time	UTC	UTC + correction (-5s)
09h02m06s	10h02m06s	10h02m01s
10h03m15s	11h03m15s	11h03m10s
11h00m00s	12h00m00s	11h59m55s

Comments

According to the initial position the navigation zone is the Azores archipelago.

Its time zone is GMT-1. The time zones are tabulated in the Nautical Almanac.

We use the local time as deck time, this avoids time lags. For computations however we need to convert the deck time to UTC. and also to correct the chronometer error.

The Dip is the sextant altitude i.e. 1,7+1,3=3m.

Compute the successive positions with traverse tables

Entrance in table		
Distance	**Rv**	**lm**
5 M	212°	35°

Results from table		
Δl	**Dep**	**Δg**
4,2'(-)	2,6'	3,2'(+)

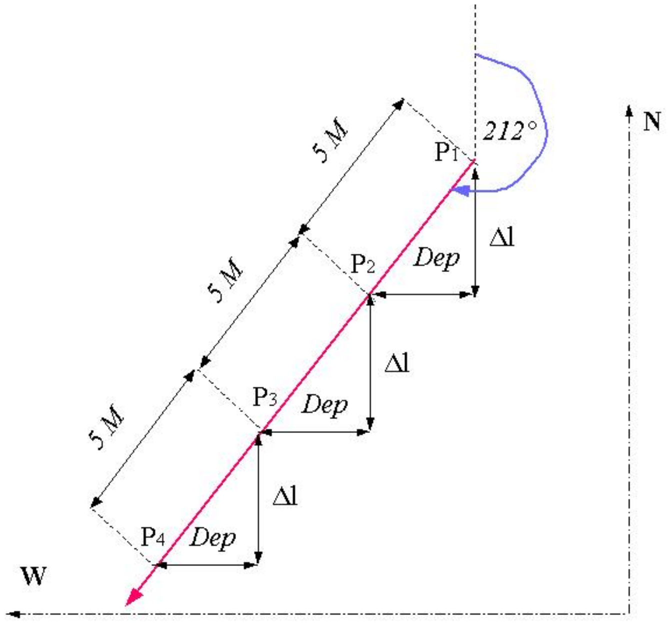
Position	Latitude	Longitude	At Deck Time
P1	35° 19,0' N	15° 17,2' W	09h00
P2	35° 14,8' N	15° 20,4' W	10h00
P3	35° 10,6' N	15° 23,7' W	11h00
P4	35° 06,3' N	15° 26,9' W	12h00

Comments:

As our speed is 5 kn the distance covered each hour is 5 miles. We use the traverse tables because our distances are very short. We consider that each distance is then the hypotenuse of the plane right angled triangle with sides Dep and Δl.

The traverse tables are nothing else than the tabulation of the formulas for right angled triangles:

Δl = (dist x cos Rv) (Δl is a distance and an angle)
Dep = (dist x sin Rv) (Dep is a distance but not an angle)
Δg = Dep/cos lm (Δg is not a distance but an angle, lm is the main latitude)



We see on the figure that the latitude is decreasing and the longitude increasing.

Determine the basic parameters

GHA☉	Increments	GHA☉	UTC
332°35,2’	30,3’	333°05,5’	10h02m01s
347°35,4’	47,5’	348°22,9’	11h03m10s
347°35,4’	14°58,8’	362°34,2’	11h59m55s

GHA☉	g(W)=-	LHA	UTC
333°05,5’	15°17,2’	317°48,3’	10h02m01s
348°22,9’	15°20,4’	333°02,5’	11h03m10s
362°34,2’	15°23,7’	347°10,5’	11h59m55s

δ	Increment d=1,0	δ	UTC
3°17,4’S	0,0’	3°17,4’S	10h02m01s
3°18,3’S	0,1’	3°18,4’S	11h03m10s
3°18,3’S	1,0’	3°19,3’S	11h59m55s

The basic parameters

l	δ	LHA
35°19,0’N	3°17,4’S	317°48,5’
35°14,8’N	3°18,4’S	333°02,5’
35°10,6’N	3°19,3’S	347°10,8’

Comments

The values GHA and δ are found in the nautical almanac.

In daily pages section we find the values GHA, δ for each entire hour. The d correction is found on the bottom of each daily page.

The yellow pages of the Nautical Almanac give the increments and corrections we have to add to the entire hour value.

Ex. : GHA at 11h00m00s is 347°35,4′; the increment for 59min05s is 14°58,8′

δ at 11h00m00s is 3°18,3′S the increment for 59min and d=1,0 is 1,0′.

Note that δ is slow value that is incremented per minute and GHA a fast value that is incremented per second. Remember increments always have a positive sign and are always added.

The LHA = GHA + g, accordingly to the sign rule g is negative because it is west.

Computation scheme for Hc, Az, Hv and ΔH

The used formulas are the haversine formula in logarithmic shape and the ABC formulas. θ is only a auxiliary variable.

A shaded element is the algebraic sum of the elements above, except for element (*)

All values are found in Nories tables.

log cos l	
log cos δ	
log hav LHA	
log hav θ	

l	
δ	
l-δ	
hav (l-δ) (*)	

hav θ	
hav(l-δ)	
hav (90°-h)=hav ζ	

hc	90°-ζ	

A		
B		
C		
Zn		

hm	
i	
dip	
n	
hv	

Δh	hc-hm	

All you have to do is to look up the values in Nories tables and eventually perform an interpolation. Then add them up

First Position

log cos 35° 19,00’	$\bar{1}$,91167
log cos 3° 17,40’	$\bar{1}$,99928
log hav 317° 48,30’	$\bar{1}$,11250
log hav θ	$\bar{1}$,02345

l	35°19,0’
δ	03°17,4’
l-δ	38°36,4’
hav (l-δ)	0,10928

hav θ	0,10555
hav(l-δ)	0,10928
hav (90°-h)	0,21483

hc	90°-55°13,6’	34°46,4’

A	0,78 S	
B	0,08 S	
C	0,86 S	
Zn	54,2° S E	E because LHA 180<LHA<360

hm	34° 35,0’
i	2,0’
dip	-3,1’
n	14,6’
hv	34° 48,5’

Δh	hc-hm	34°46,4-34°48,5’= -2,1’ (to)

Second Position

log cos 35° 14,8’	$\bar{1}$,91205
log cos 3° 18,4’	$\bar{1}$,99928
log hav 333° 02,5’	$\bar{2}$,73505
log hav θ	$\bar{2}$,64638

interpolation

l	Δ	Log cos l	Δ
35°14,0’		$\bar{1}$,91212	
35°14,8	80	$\bar{1}$,91205	7
35°15,0	100	$\bar{1}$,91203	9

l	35°14,8’
δ	03°18,4’
l-δ	38°33,2’
hav (l-δ)	0,10898

hav θ	0,04430
hav(l-δ)	0,10899
hav (90°-h)	0,15329

hc	90°-46°05,8’	43°54,2’

A	1,39 S	
B	0,13 S	
C	1,52 S	
Zn	38,8 SE	E because LHA 180<LHA<360

hm	43° 42,3’
i	2,0’
dip	-3,1’
n	15,2’
hv	43°56,4’

Δh	hc-hm	43°54,2-43°56,4’= -2,2’ (to)

Third Position

log cos 35° 10,6’	$\bar{1}$,91243
log cos 3° 19,3’	$\bar{1}$,99927
log hav 347° 10,5’	$\bar{2}$,09599
log hav θ	$\bar{2}$,00769

interpolation

l	Δ	log cos l	Δ
35°10,0’		$\bar{1}$,91248	
35°10,6	60	$\bar{1}$,91243	5
35°11,0	100	$\bar{1}$,91239	9

l	35°10,6’
δ	03°19,3’
l-δ	38°29,9’
hav (l-δ)	0,10869

hav θ	0,01018
hav(l-δ)	0,10869
hav (90°-h)	0,11887

hc	90°-40°20,1’	49°39,9’

A	3,09 S	
B	0,26 S	
C	3,35 S	
Zn	20 SE	E because LHA 180<LHA<360

hm	49° 27,6’
i	2,0’
dip	-3,1’
n	15,2’
hv	49°41,7’

Δh	hc-hm	49°39,9’-49°41,7’= -1,8’(to)

Fourth position at Meridional passage

Mer. Pass.	11h50m00s
ε	10m22s
Culmination at 0°W	12h00m22s
Arc to time of 15°26,4W	01h01m46s
Culmination time	13h02m08s

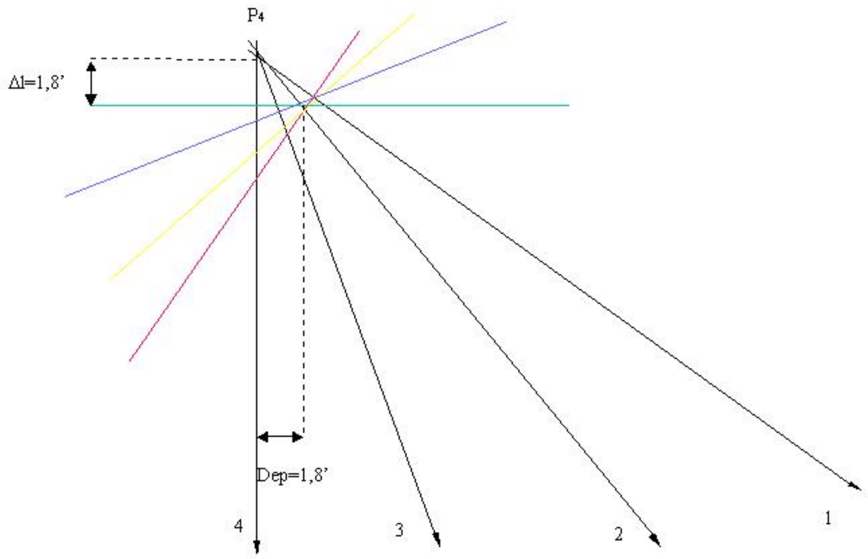
δ	03°20,3’S
d=1,0	0,0’
δ	03°20,3’S
l	35°06,3’N
ζ=|l–δ|	38°26,6’

hm	51° 20,8’
i	2,0’
dip	-3,1’
n	15,2’
hv	51°34,9’

hc=90°–ζ	51°33,4’

Δh	hc-hv	51°33,40’-51°34,9’=-1,5’(to)

Plotting the position



The figure is not to scale

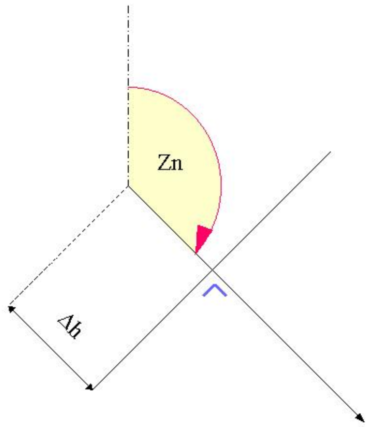
The correction to apply: 1,8 S ; 2,2 E
35°06,3'-1,8'=35°4,5'
15°26,9'-2,2'=15°24,7

35°04,5' N

15°24,7' W

Comment:

From the final position P_4 draw a line for each obtained Δh and Zn at an angle Zn. Terminate it with an arrow. If Δh is "towards" then draw a perpendicular at a distance Δh in the direction of the arrow.



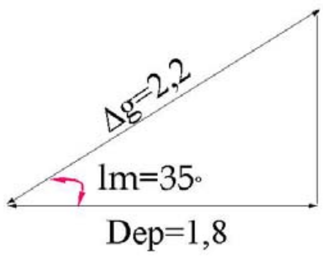
The scale you choose for Δh is the scale with which you must measure Dep and Δl on the plot.

Example:

You draw a Δh of 2′ at a distance of 1 cm and then you measure a Dep of 0,9 cm and a Δl of 1,2 cm then your Dep is 1,8′ and your Δl is 2,4′.

Construct Δg

Construct a right angled triangle with base equal to Dep and angle lm, the hypotenuse then is Δg.



This could also be done with the traverse tables however this is less accurate.

PROBLEM 2

On the tenth of July 2001 a boat is steering a course of 60° at a speed of 5 knots.

Her estimated position at 10 am deck time is 19° 56'S; 60° 07,0'E.

The sextant was hold at 3 m above the waterline. The chronometer is running 3s too fast. The deck time is directly derived from the chronometer time. The sextant has an index error of -5 minutes.

The following altitudes of the sun were noted at the respective deck times :

10h00m00s	*37° 53,1'*
11h00m29s	*44° 55,2'*
Meridional passage	*47° 42,7'*

Determine the position of the boat at meridional passage.

Summarised data

DATE	Time Zone	Dip	Rv	speed	c	i	l_1	g_1
10/07/2001	GMT+4	3m	60°	5kn	+3s	-5'	19° 56,0'S	60° 07,0' E

Observation at Local Time	Measured height
10h00m00s	37° 53,1'
11h00m29s	44° 55,2'
Meridional pass. = +/-12h00	47° 43,8'

Local Time	UTC	UTC + correction (+3s)
10h00m00s	6h00m00s	6h00m03s
11h00m29s	7h00m29s	7h00m32s

Comment

According to the initial position the navigation zone is around Mauritius.

Its time zone is GMT+4.

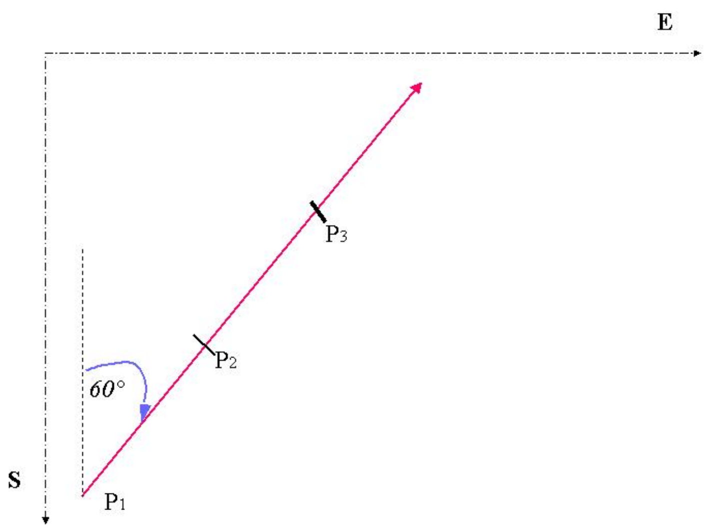
The successive positions

Entrance in table		
Distance	**Rv**	**lm**
5 M	60°	20°

Results from table		
Δl	**Dep**	**Δg**
2,5'(-)	4,3'	4,6'(+)

Position	Latitude	Longitude	UTC
P1	19° 56,0' S	60° 07,0' E	6h00m03s
P2	19° 53,5' S	60° 11,6' E	7h00m32s
P3	19° 51,0' S	60° 16,2' E	M.P.

Sketch



We see on the figure that the latitude is decreasing and the longitude increasing.

Determine the basic parameters

GHA☉	Increments	GHA☉	UTC
268°40,2’	0,8’	268°41,0’	6h00m03s
283°40,1’	8,0’	283°48,0’	7h00m32s

GHA☉	g(E)=+	LHA	UTC
268°41,0’	60°07,0’	328°48,0’	6h00m03s
283°48,0’	60°11,6’	343°59,6’	7h00m32s

δ	Increment d=0,3	δ	UTC
22°13,3’N	0,0’	22°13,3’N	6h00m03s
22°12,9’N	0,0’	22°12,9’N	7h00m32s

The basic parameters

l	δ	LHA
19°56,0’S	22°13,3’N	328°48,0’
19°53,5’S	22°12,9’N	343°59,6’

First Position

log cos 19°56,0’	1,97317
log cos 22°13,3’	1,96648
log hav 328°48,0’	2,85925
log hav θ	2,79890

l	19°56,0’
δ	22°13,3’
l-δ	42°09,3’
hav (l-δ)	0,12933

hav θ	0,06293
hav(l-δ)	0,12933
hav (90°-h)	0,19226

hc	90°-52°00,8’	37°59,2’

A	0,60N	
B	0,78N	
C	1,38N	
Zn	37,6°NE	E because 0°<LHA<180°

hm	37°53,1’
i	-5’
dip	-3,1’
n	+14,8’
hv	37°59,8’

Δh	hc-hm	37°59,2’-37°59,8’=-0,6 towards

Second Position

log cos 19°53,5’	1,97328
log cos 22°12,9’	1,96650
log hav 343°59,6’	2,28747
log hav θ	2,22725

l	19°53,5’
δ	22°12,9’
l-δ	42°06,4’
hav (l-δ)	0,12905

hav θ	0,01687
hav(l-δ)	0,12905
hav (90°-h)	0,14592

hc	90°-44°54,9’	45°5,1’

A	1,27N	
B	1,48N	
C	2,75N	
Zn	21,2NE	E because 0°<LHA<180°

hm	44°55,2’
i	-5’
dip	-3,1’
n	+15,0’
hv	45°2,1’

Δh	hc-hm	45°5,1-45°2,1’ = 3,0 away

Third position at Meridional passage

Mer. Pass.	12h05m00s
ε	-00h05m21s
Culmination at 0°W	11h59m39s
Arc to time of 60°12,6'E	-4h01m04s
Culmination time UTC	7h58m35

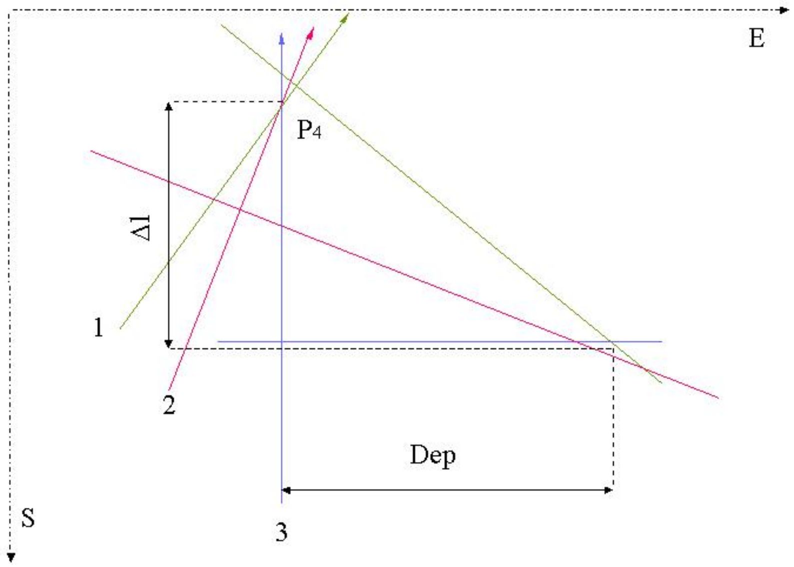
δ	22°12,9'
d=0,3	-0,3'
δ	22°12,6'
l	+19°51,0'
ζ=|l–δ|	42°03,6'

hm	47°42,7'
i	-5'
dip	-3,1'
n	15,1'
hv	47°49,7'

hc=90°–ζ	47°56,4'

Δh	hc-hv	47°56,4'-47°49,7'=6,7' away

Plot



Position at Meridional passage :
The correction to apply: 7′ S ; 10,9′ E
19°51,0′+ 7,0′=35°04,5′
60°16,2′+10,9′=60°27,1′

35°04,5′
60°27,1′

PROBLEM3

On 8 March 2001 a boat is steering a course of 70° at a speed of 1,8 knots.

Her estimated position at 10h30 UTC is 35° 18'N; 15°15° 07,0'E. We assume that this estimated position is quite exact.

The sextant was hold at 3 m above the waterline. The chronometer is exactly matched on UTC. The sextant has no index error.

The following altitudes of the sun were noted at the UTC :

10h30m00s	*34°39,3'*
Culmination height	*49° 44,8'*

Determine the position at culmination time with the Pagel method..

Summarised data

DATE	Time Zone	Dip	Rv	speed	c	i	l_1	g_1
8/03/2001	GMT	3m	70°	1,8kn	0s	0'	35° 18,0'N	15° 15' W

Observation at UTC	Measured height
10h30m00s	34° 39,3'
At $\pm$ 13h00	49° 44,8'

Comment

According to the initial position the navigation zone is around Madeira which is keeping Greenwich Meantime. The Pagel method can be used only if the estimated position is quite close to the real position. These positions are considered as being close when their respective azimuths are nearly equal.

Basic parameters for estimated position Pe at 10h30

GHA☉	Increments	GHA☉	UTC
327°18,4’	7°30’	334°48,4’	10h30m00s

GHA☉	g(W)=-	LHA	UTC
334°48,4’	15°15,0’	319°33,4’	10h30m00s

δ	Increment d=1,0	δ	UTC
4°47,1’S	-0,5’	4°46,6’S	10h30m00s

l
35°18,0’N

Hc and Zn at 10h30

log cos 35°18,0’	$\underline{1}$,91176
log cos 4°46,6’	$\underline{1}$,98849
log hav 319°33,4’	$\underline{1}$,07728
log hav θ	$\underline{2}$,98753

l	35°18,0’
δ	4°46,6’
l-δ	40°04,6’
hav (l-δ)	0,11741

hav θ	0,09717
hav(l-δ)	0,11741
hav (90°-h)	0,21458

hc	90°-55°11,4’	34°48,6’

A	0,82 S	
B	0,12 S	
C	0,94 S	
Zn	52° SE	E because LHA 180<LHA<360

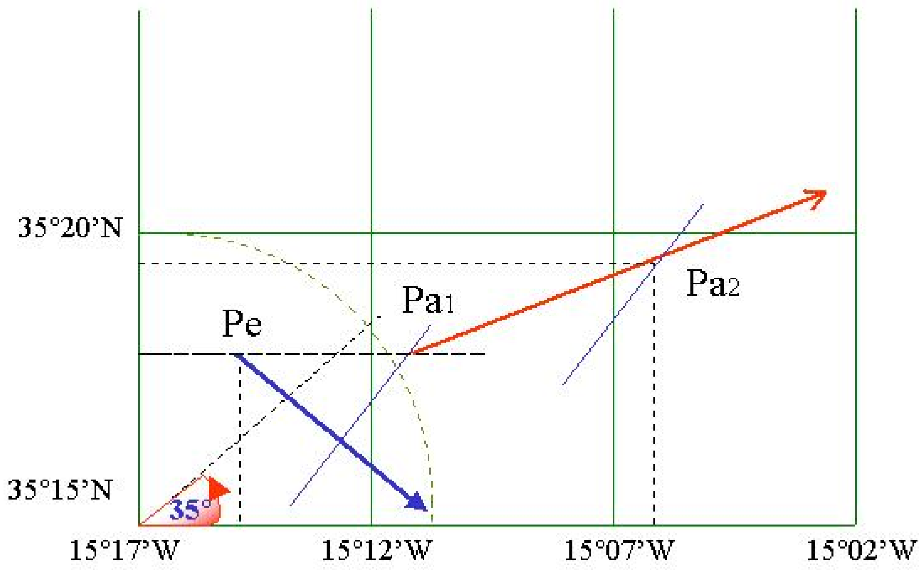
hm	34°39,3’
i	0’
dip	-3,1’
n	14,7’
hv	34°50,9’

Δh	hc-hm	34°48,6’-34°50,9’=-2,3 towards

Comment

This is a classic Hc, Zn and Δh computation.

Plot the position



On the plot we find Pa1 and Pa2

Position	Latitude	Longitude	UTC
Pe	35° 18,0' N	15° 15,0' W	10h30m00s
Pa1	35° 18,0' N	15° 11,4' W	10h30m00s
Pa2	35° 19,5' N	15° 06,2' W	M.P.

The Pagel Method

Construct a middle latitude chart and plot the estimated position Pe. With the found Azimuth and intercept draw the fix of Pe. The adjusted Position Pa1 lays on the intersection between the parallel of Pe and its fix. The Position Pa2 is obtained by transferring the fix over a distance of 4,5 nM under 70°. (as the distance covered in 2,5 hours is : 1,8x2,5=4,5).All we do in fact is adapting the longitude of our estimated position to the longitude of Pa1. The intercept remains the same because we remain on the fix as the fix is the collection of points having the same intercept at that moment, but different Azimuths. However the change in Azimuth is a slow value.

Culmination Height of Pa2

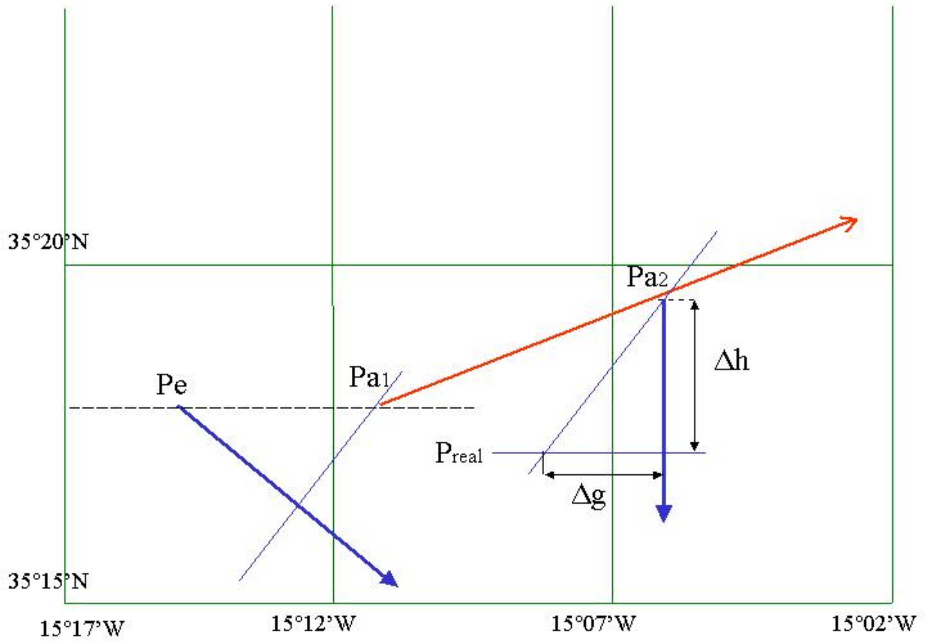
Mer. Pass.	12h11m00s
ε	10m45s
Culmination at 0°W	12h00m15s
Arc to time of 15°6,2'W	1h00m24s
Culmination time UTC	13h00m39s

δ	-4°44,1'
d=1,0	0'
δ	4°44,1
l	35°19,5'
ζ=|l–δ|	40°03,6'

hm	49°44,8'
i	0'
dip	-3,1'
n	16,7'
hv	49°58,4'

hc=90°–ζ	49°56,4'

Δh	hc-hv	49°56,4'-49°58,4'=2,0'towards



We find by computation that Δl=2′ towards and Δg = Δlx C= 2 x 0,92 = 1,88′

C is the parameter from in the ABC tables for Zn computation.

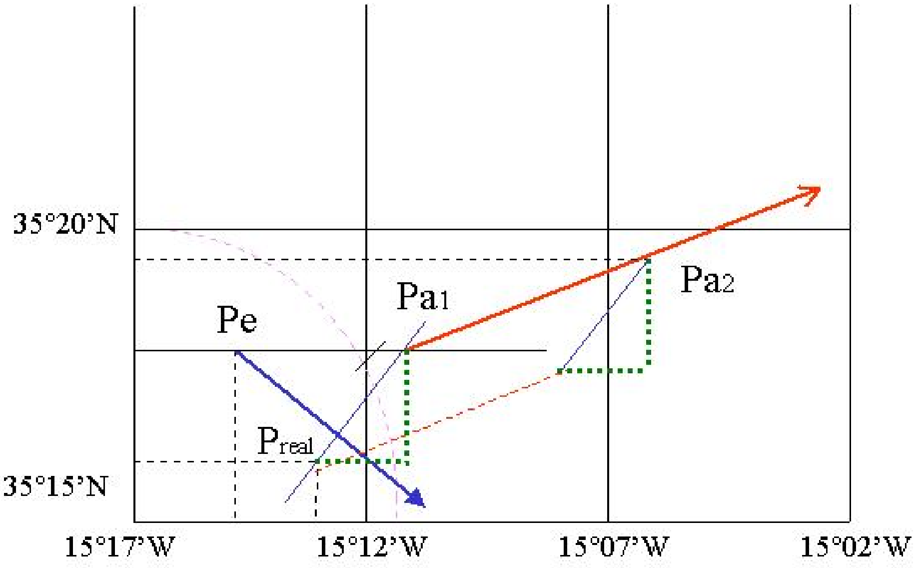
According to the plot Δg has to be applied westwards.

Which gives : 35°19,5′-2,0′=35°17,5′N and 15°4,2W+1,88′=15°6,1′

35°17,5′N

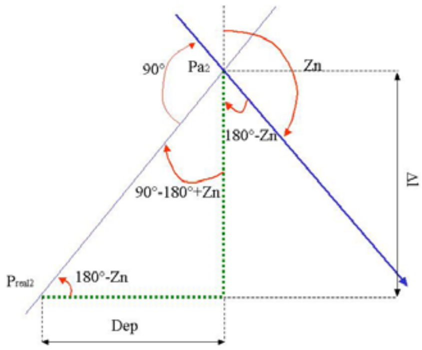
15°04,2W′

Verification



Position	Latitude	Longitude	UTC
Preal1	35° 16,0' N	15° 13,3' W	10h30m00s
Preal2	35°17,5'N	15° 08,1'W	M.P.

By running back Preal2 we find our real initial position Preal1 at 10h30. We can verify that Preal1 is laying on the fix of 10h30.



The factor C: on the figure above is a detail of the plot we see that Dep = Δl x cotg Zn As Δg= Dep x sec l then $\Delta g = \Delta l$ x cotg Zn sec l = Cx Δl. As C=See definition of C in formularies.

PROBLEM4
Distance and Course Computation with Nories Table

Belem(Brazil)	*01°27'S*
	48°30'W

Brest(France)	*48°23'N*
	04°29'W

ORTHODROMIC TRACK

l1	*48°23'*
l2	*+01°27'*
Δl	*49°50'*

g2	*48°30'*
g1	*-04°29'*
Δg	*44°01'*

log cos *01°27'*	*1,99986*
log cos *48°23'*	*1,82226*
log hav *44°01'*	*1,14746*
log hav θ	*2,96958*

Hav θ	*0,09323*
Hav *49°50'*	*0,17749*
Hav M	*0,27072*

M	*62°43'*
M	*62x60+43*
M in nM	*3763 nM*

LOXODROMIC TRACK

Δl	*49x60+50*	*2990 nM*
Δg	*44x60+1*	*2641 nM*

lc1	*86,42*
lc2	*+3308,52*
Δlc	*3394,94*

log Δlc	*3,53083*

colog Δlc	*-3,53083*

Integer *4*	*4,00000*
colog Δlc	*-3,53083*
colog Δlc	*4,46917*

log Δg	*3,42177*
colog Δlc	*4,46917*
log tan Rv	*1,89094*

Course from Belem to Brest			**Course from Brest to Belem**	
Rv	*37°53'*	*+180°*	**Rv**	*217°53'*

log sec Rv	*0,10278*
log Δ l	*3,47567*
log M	*3,57845*

M	*3788 nM*

PROBLEM 5

Date: 6th july of 2001

Speed: immobile dip=2,8m i=0,0’ c= 0 s

Position: 42°15,1’N zone : western Mediterranean

6°25,0’E

Determine the position of the boat

Data Sample from the Nautical Almanach

Lat.	Sunset	Twilight Civil	Twilight Naut.
°	h m	h m	h m
N 72	□	□	□
N 70	□	□	□
68	□	□	□
66	23 08	////	////
64	22 14	////	////
62	21 42	////	////
60	21 19	22 48	////
N 58	21 00	22 10	////
56	20 45	21 44	////
54	20 32	21 24	22 54
52	20 20	21 07	22 19
50	20 10	20 53	21 55
45	19 49	20 25	21 13
N 40	19 31	20 04	20 45
35	19 17	19 46	20 23
30	19 05	19 32	20 05

Tabular Interval 10°	Tabular Interval 5°	Tabular Interval 2°	Difference between the times for consecutive latitudes 5m	10m	15m	20m	25m	30m
° ′	° ′	° ′	m	m	m	m	m	m
0 30	0 15	0 06	0	0	1	1	1	1
1 00	0 30	0 12	0	1	1	2	2	3
1 30	0 45	0 18	1	1	2	3	3	4
2 00	1 00	0 24	1	2	3	4	5	5
2 30	1 15	0 30	1	2	4	5	6	7
3 00	1 30	0 36	1	3	4	6	7	8
3 30	1 45	0 42	2	3	5	7	8	10
4 00	2 00	0 48	2	4	6	8	9	11
4 30	2 15	0 54	2	4	7	9	11	13
5 00	2 30	1 00	2	5	7	10	12	14

Compute Civil twilight (CVT)

	20h04m00s
Data from table	+ 09m00s
At 0°E	**20h13m00s**
Arc to time(6°25’)	- 25m40s
UTC	**19h47m20s**
shift	+ 2h00m00s
CVT at LMT	**21h34m40s**

At approximately this decktime we will measure the selected stars

Compute LHAγ at local civil twilight

	209°49,3’
I (34m)	8°40,0’
GHAγ (19h34m)	**218°29,3’**
+g(E)	6°25,0’
LHAγ (19h34m)	**224°54,3’**

Use as entry LHAγ 224° and 42°N in the HO249 tables and find the selected stars.

Sample from HO249 table

LAT 42°N

LHA ♈	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn	Hc	Zn
			VEGA		Rasalhague		ANTARES		ARCTURUS		REGULUS		Dubhe	
220	28 10	053	45 39	073	42 00	115	17 01	154	66 35	194	24 22	264	52 46	321
221	28 45	053	46 22	074	42 40	116	17 20	155	66 23	197	23 38	265	52 18	321
222	29 21	054	47 05	074	43 20	117	17 38	156	66 10	199	22 54	266	51 50	321
223	29 57	054	47 48	075	44 00	118	17 56	157	65 54	201	22 09	266	51 21	320
224	30 34	055	48 31	075	44 39	119	18 13	158	65 37	203	21 25	267	50 53	320
	DENEB		ALTAIR		Rasalhague		ANTARES		ARCTURUS		Denebola		Dubhe	
225	31 10	055	18 43	095	45 18	120	18 29	159	65 19	206	40 43	251	50 25	320

Determine LHAγ for each star at observation time

	Deneb (1)	Arcturus (2)	Dubhe (3)
	19h30	19h32m3s	19h34m59s
GHAγ	209°49,3’	209°49,3’	209°49,3’
I	7°31,2’	8°02,1’	8°44,7’
	217°20,5’	**217°51,4’**	**218°34,0’**
g(E)	6°39,5’	6°08,6’	6°26,0’
LHAγ	**224°**	**224°**	**225°**

choose g such as the result for LHAγ is a natural number

With LHAγ as entry we find the HO249 table:

Hc=30°34’ Zn=55°	**Hc=65°37’ Zn=203°**	**Hc=50°25’ Zn=320°**

Correct the measured hights with dip and refraction coeff.

Deneb	Arcturus	Dubhe	
Hm=30°40,6’	Hm=65°19,5’	Hm=50°31,6’	Sextant heights
C1= - 1,6’	C1= - 0,4’	C1 = - 0,8’	Refraction correction
C2= - 3,0’	C2= - 3,0’	C2= - 3,0’	dip
Hv=30°36,0’	**Hv=65°22,9’**	**Hv=50°35,4’**	

Compute the intercepts

Hv	30°36,0’	**Hv**	65°19,5’	**Hv**	50°31,6’
Hc	30°34,0’	**Hc**	65°37,0’	**Hc**	50°25,0’
Δh1	**2,0’**	**Δh2**	**- 17,5’**	**Δh3**	**6,6’**

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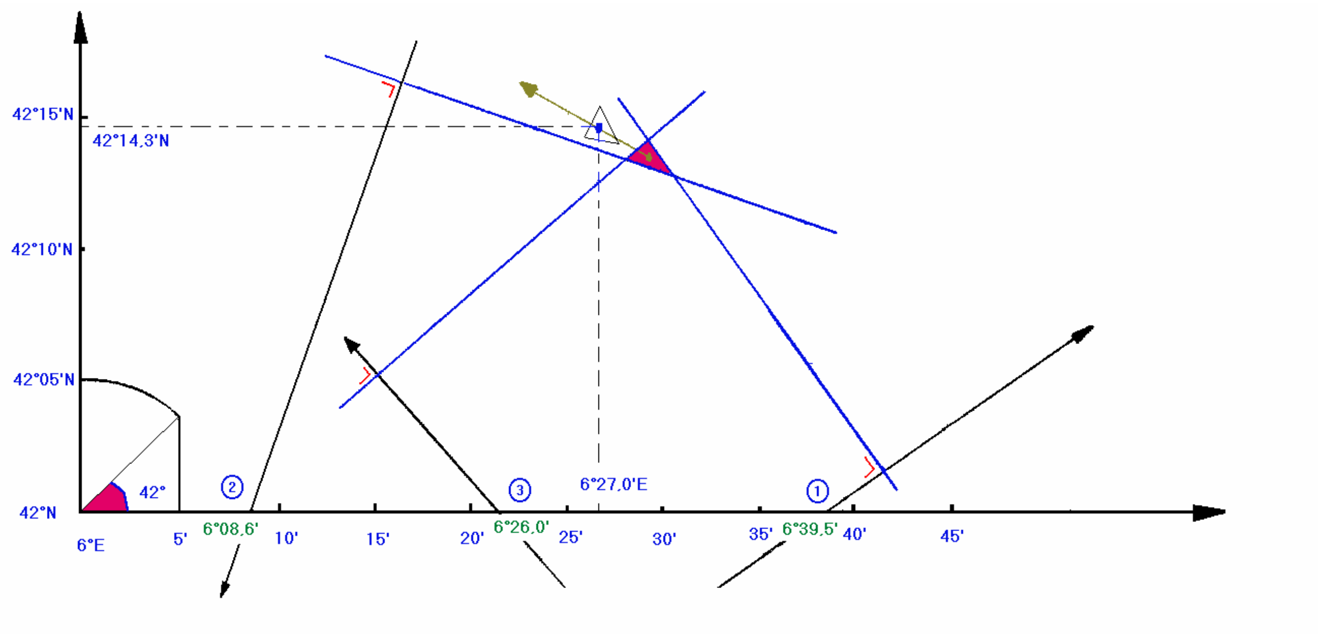
Plot

Plot respective fixes in the three estimated positions 6°39,5'E, 6°08,6'E, 6°26,0'E all at 42°N



Transfer the position triangle over 2,2 M at 301° in order to correct the error due to nutation of the earth's axis.

CORRECTION FOR PRECESSION AND NUTATION

LHA ♈	North latitudes							South latitudes						LHA ♈
	N 80°	N 70°	N 60°	N 50°	N 40°	N 20°	0°	S 20°	S 40°	S 50°	S 60°	S 70°	S 80°	
							2001							
°	′ °	′ °	′ °	′ °	′ °	′ °	′ °	′ °	′ °	′ °	′ °	′ °	′ °	°
0	1.4 201	1.6 218	1.9 229	2.3 236	2.6 240	3.0 245	3.2 247	3.1 245	2.6 241	2.3 236	2.0 229	1.6 219	1.4 203	0
30	1.6 225	1.9 235	2.3 241	2.6 245	2.9 247	3.2 249	3.2 249	2.8 246	2.2 239	1.8 232	1.5 220	1.2 201	1.1 175	30
60	1.7 247	2.1 252	2.5 255	2.8 256	3.0 257	3.2 258	3.0 257	2.5 255	1.7 247	1.2 238	0.8 218	0.7 178	0.9 139	60
90	1.8 269	2.2 269	2.6 269	2.9 270	3.1 270	3.2 270	2.9 270	2.3 269	1.4 269	0.9 268	0.4 266	0.2 097	0.7 092	90
120	1.7 291	2.2 287	2.5 284	2.8 283	3.0 282	3.2 281	3.0 282	2.5 285	1.7 292	1.2 301	0.8 321	0.6 004	0.9 044	120
150	1.6 313	2.0 304	2.3 298	2.6 295	2.9 292	3.2 290	3.1 290	2.8 293	2.1 301	1.8 308	1.4 321	1.2 341	1.1 007	150
180	1.4 337	1.6 321	2.0 311	2.3 304	2.6 299	3.1 295	3.2 293	3.0 295	2.6 300	2.3 304	1.9 311	1.6 322	1.4 339	180
210	1.1 005	1.2 339	1.5 320	1.8 308	2.2 301	2.8 294	3.2 291	3.2 291	2.9 293	2.6 295	2.3 299	1.9 305	1.6 315	210
240	0.9 041	0.7 002	0.8 322	1.2 302	1.7 293	2.5 285	3.0 283	3.2 282	3.0 283	2.8 284	2.5 285	2.1 288	1.7 293	240
270	0.7 088	0.2 083	0.4 274	0.9 272	1.4 271	2.3 271	2.9 270	3.2 270	3.1 270	2.9 270	2.6 271	2.2 271	1.8 271	270
300	0.9 136	0.6 176	0.8 219	1.2 239	1.7 248	2.5 255	3.0 258	3.2 259	3.0 258	2.8 257	2.5 256	2.2 253	1.7 249	300
330	1.1 173	1.2 199	1.4 219	1.8 232	2.1 239	2.8 247	3.1 250	3.2 250	2.9 248	2.6 245	2.3 242	2.0 236	1.6 227	330
360	1.4 201	1.6 218	1.9 229	2.3 236	2.6 240	3.0 245	3.2 247	3.1 245	2.6 241	2.3 236	2.0 229	1.6 219	1.4 203	360